SEQUENCE & SERIES

Some Terms
• $a_1, a_2, a_3 \ldots a_n$ are terms or elements of sequence. • T_n : represents the nth term • S_n : represents the sum of series of n terms
Remember, $S_n - S_{n-1} = T_n$

Arithmetic Progression (A.P.)

Series : $a, a + d, a + 2d \ldots\ldots \underbrace{a + (n-1)d}_{\text{nth term}}$

nth odd number	2n-1
last term (l)	$[a + (n-1)d]$
Sum of series	• $S_n = (n/2)\,[2a + (n-1)d]$ • $S_n = (n/2)\,[a + l\,]$

- Common terms of common difference d_1 & d_2 are also in AP with common difference of $LCM\{d_1, d_2\}$
- Any operation carried (+ , − , x , /) turns an AP into another AP
- Sum of the terms equidistant from beginning and end is same
- **3 term AP** : a-d, a, a+d
- **4 term AP** : a-3d, a-d, a+d, a+3d
- **5 Term AP** : a-2d, a-d, a, a+d, a+ 2d

Assuming terms

- If $T_n = an + b$, series is AP, **C.D. = a**
- If $S_n = an^2 + bn + c$, series is AP, **C.D. = 2a**
- Sum of **infinite terms** in AP is ∞ $(d > 0)$ & $-\infty$ $(d < 0)$.
- If pth term is q and the qth term is p, then the **mth term** is $= p + q - m$.
- If Sum of p terms is q and sum of q terms is p, then the **sum of (p + q) terms** is $-(p + q)$
- If Sum of p terms is equal to the sum of q terms, then the **sum of (p +q) terms** is zero

Geometric Progression (G.P.)

Series : $a, ar, ar^2, ar^3 \ldots\ldots \underbrace{ar^{n-1}}_{\text{nth term}}$

r is common ratio

nth term (a_n)	ar^{n-1}
Sum of series	$S_n = \left(\dfrac{a(r^n - 1)}{r-1}\right) = \left(\dfrac{T_{n+1} - a}{r-1}\right)$
Provided |r| < 1 & n→∞	$S_\infty = \dfrac{a}{1-r}$

- **3 term AP** : $a/r, a, ar$
- **4 term AP** : $a/r^3, a/r, ar, a/r^2$
- **5 Term AP** : $a/r^2, a/r, a, ar, ar^2$
- Multiplying or dividing with a constant produces GP
- Ratio of terms of two GP's Produces another GP
- Terms of GP raised to a power also gives GP

Important Points

- if a,b,c are AP : $2b = a+c$
- if a,b,c are GP : $b^2 = ac$
- No term in GP can be Zero

DON'T FORGET!

Harmonic Progression (H.P.)

if Reciprocals of terms are in AP

$a_1, a_2, a_3 \ldots a_n$ are in H.P.	$1/a_1, 1/a_2, 1/a_3 \ldots 1/a_n$ are in AP

- if a,b,c are in H.P. then, $b = \dfrac{2ac}{a+c}$

Arithmetic Mean (A.M.)

- Arithmetic mean of two numbers a & b is $(a+b)/2$
- A.M. of n numbers : $(a_1 + a_2 + a_3 \ldots + a_n)/n$
- To **insert 'n' AM's** between a & b to form an AP with nth term as $A_n = a + nd$
  - **Common difference** : $d = \dfrac{b-a}{n+1}$
- Sum of n A.M's inserted b/w a & b $=$ **$n(a+b)/2$**

Geometric Mean (G.M.)

- a and b are two real numbers of the same sign and G is the G.M. between them, $G^2 = ab$
- GM doesn't exist with opposite signs a & b
- To **insert 'n' GM's** between a & b to form a GP with nth term $G_n = ar^n$
  - **Common ratio :** $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$
- Product of n G.M's inserted b/w a & b = $\mathbf{(G)^n}$ where $\mathbf{G = \sqrt{(ab)}}$ i.e. GM between a & b

Harmonic Mean (H.M.)

- If H is the HM between a and b, then a, H, c are in H.P. where H = 2ac/(a+c)
  - **Common Difference :** $d = \frac{a-b}{ab(n+1)}$
- Sum of n H.M's inserted bw a & b is
  - **n (1/H) = n (a+b)/2ab**

Relation Between A.M. , G.M. & H.M.

- For any +ve numbers : **AM ≥ GM ≥ HM**
  - equality holds when a = b
- Use the relation : **AM ≥ GM** to find minimum value in a given question
- Also, $G^2 = A.H$
- The quadratic equation with roots a & b can be written as : $\mathbf{x^2 - 2Ax + G^2 = 0}$ and in case of Cubic equation : $\mathbf{x^3 - 3Ax^2 + (3G^3/H)x - G^3 = 0}$

Sigma (summation) Notation

e.g. $\sum_{r=1}^{n} r^a$ or $\sum n^a = 1^a + 2^a + 3^a + ... + n^a$

Properties of Sigma Notation

(i) $\sum_{i=l}^{k} a = a + a + a...$ (k times) $= ka$

(ii) $\sum_{i=l}^{k} ai = a\sum_{i=l}^{k} i$

(iii) $\sum_{i=i_0}^{i_n} \sum_{j=j_0}^{j_n} a_i a_j = \sum_{j=j_0}^{j_n} \sum_{i=i_0}^{i_n} a_i a_j$

(iv) $\sum_{r=l}^{n} (a_r \pm b_r) = \sum_{r=l}^{n} a_r \pm \sum_{r=l}^{n} b_r$

Pi (Multiplication) Notation

e.g. $\prod_{n=1}^{k} n^m = 1^m \times 2^m \times 3^m \times 4^m \times .... \times k^m$

Method of Difference

- **Step I:** Denote the nth term by Tn and the sum by Sn.
- **Step II:** Rewrite the given series with each term shifted by one place to the right.
- **Step III:** By subtracting the later series from the former, find Tn.
- **Step IV:** From Tn, Sn can be found by appropriate summation.